Task sheet 4

Task 24. The $\exists^*$-theory of word equations consists of all sentences of the form:

$$\exists_{x_1,x_2,\ldots,x_k} \varphi(x_1, x_2, \ldots, x_k)$$

where φ is quantifier-free logic formula that uses $\wedge, \vee, \neg$ as connectives and atomic formulas that are word equations that use constants from Σ^* and variables $x_1, x_2, \ldots, x_k$.

Show that we can verify sentences from this theory in PSPACE.

Hint: The algorithm will heavily employ non-determinism to reduce this case to a system of word equations. The inequalities are easy to handle: look for first differences.

Task 25. Show that a positive theory of word equations over free semigroup is undecidable. Two alternations of quantifiers are enough (one, if you put some thought into it: $\forall^*\exists^*$ is undecidable).

Hint: First make the claim about the whole theory and then eliminate the negation as in Task 24.

Task 26. Consider the positive $\exists^*\forall^*$ fragment of word equations over a semigroup (no negation). Show that it is decidable.

To this end consider first the $\forall^*$ positive fragment.

Hint: Universally quantified equations tend to be false.