Task sheet 5

Task 27. Show that for large enough r_i there is a set of enough random string.

Hint: The simplest proof is through Kolmogorov's complexity, but random strings should also be good.

Task 28. Show that each of the defined rewriting systems P_i is confluent and thus each term has a unique normal form (note that the rewriting system is length-reducing).

Task 29. Let $\mathbb{G}_1, \ldots, \mathbb{G}_m \leq \mathbb{G}$ be groups, $\vec{\mathbf{z}} \in \mathbb{G}$ be elements of $\mathbb{G}$ and let $i : \mathbb{G} \to \mathbb{G}$ be an automorphism of $\mathbb{G}$ such that $i(\mathbb{G}_j) = \mathbb{G}_j$. Show that

$$\exists Y_1 \in \mathbb{G} \exists Y_2 \in \mathbb{G}_2 \ldots \exists Y_m \in \mathbb{G}_m \varphi(Y_1, \ldots, Y_m, \vec{\mathbf{z}})$$

holds if and only if

$$\exists Y_1 \in \mathbb{G}_1 \exists Y_2 \in \mathbb{G}_2 \ldots \exists Y_m \in \mathbb{G}_m \varphi(Y_1, \ldots, Y_m, i(\vec{\mathbf{z}}))$$

holds.

Task 30. Let $\mathbb{G} = \langle c_1, \ldots, c_m \rangle$ be a free group and consider $h : \mathbb{G} \to \mathbb{G}$ defined by

$$\begin{aligned} h(c_1) &= g c_1 g' \\ h(c_i) &= c_i \qquad\qquad \text{for } i > 1 \ , \end{aligned}$$

where $g, g' \in \langle c_2, \ldots, c_m \rangle$. Show that h is an automorphism of $\mathbb{G}$ (so an isomorphism from $\mathbb{G}$ to $\mathbb{G}$).