Task sheet 9

Task 54. Run the algorithm for quadratic word equation on

$$abXcY = YcXba$$

Draw the resulting graph of equations considered by the algorithm.

Task 55. Show that the algorithm for quadratic equations in fact yields a description of all solutions of such na equation.

Task 56. Consider a restricted class of word equations satisfying the following two conditions: regular and *oriented*: If two variables X, Y occur on both sides of the equation then they appear in the same order on both sides (i.e. if X occurs to the left of Y on the left-hand side, the same happens on the right-hand side and vice-versa).

Show that satisfiability quadratic, regular, oriented word equations is in NP.

Task 57. Extend the algorithm for quadratic word equations so that it also allows regular constraints.

Task 58. Is it true, that the algorithm for quadratic word equations runs in (for appropriate nondeterministic choices) time polynomial in $n, \log N$, where n is the size of the instance and N the length of the length-minimal solution?

Task 59. Solve a system $\mathcal{S}_2$ of word equations in two unknowns

$$YAX = XBY$$
$$YCX = XDY$$

where $(A, B) \neq (C, D)$. That is, present a simple superset of its solutions.

By symmetry you may assume that $|s(X)| > |s(Y)|$.

Consider $|s(Y)| < |s(X)| \leq |s(Y)||A|$ and $|s(X)| > |s(Y)||A|$ separately. Inb the second case substitute $X = YAZ$.

Task 60. Solve a system $\mathcal{S}_3$ of word equations in two unknowns

$$XAY = YBX$$

To this end consider it as a quadratic equation. What can you tell about its graph?

The solution set is described as morphism applied to some words. We want an exact description, not superset.

Task 61. Solve a system $\mathcal{S}_4$ of word equations in two unknowns:

$$XAYA = YAXA \ .$$

That is, give a reasonable superset of solution.