Task sheet 10

Task 62. Show that if $u_1 w u_2 = v_1 \overline{w} v_2$ and w is reduced then either $w = 1$ or $v_1 \overline{w} \sqsubseteq u_1$ or $u_1 w \sqsubseteq v_1$, i.e. w and $\overline{w}$ cannot overlap.

Task 63. Consider the case of x being a pseudo-solution of $xu\underline{x}vx$, where $x_u = \overline{u''}$ and $x_v = w_v \overline{v'}$: (and $\overline{v''}\,\overline{w_v} \sqsubseteq x_u = \overline{u''}$); in particular, $x = \overline{u''} w_v \overline{v'}$.

Show that $x = x_u x_v = \mathrm{nf}(\overline{u''} u^{\bullet\bullet} \overline{v})$ for some $u \sqsupseteq u'' \sqsupseteq u^{\bullet\bullet}$; note that there is a reduction in $\overline{u''} u^{\bullet\bullet}$

Task 64. Let $x_u = \overline{u''} w_u$, $x_v = w_v \overline{v'}$ (in particular, $x = \overline{u''} w_u w_v \overline{v'}$) and also $x_v \sqsupseteq \overline{w_u}\,\overline{u'}$ and $\overline{v''}\,\overline{w_v} \sqsubseteq x_u$. All above are in nf; this corresponds to the main case of pseudo-solution of $xu\underline{x}vx$.

Justify the argument (especially the steps involving nf)

$$
\begin{aligned}
\overline{v''}\,\overline{w_v} &\sqsubseteq \overline{u''} w_u \\
v' v'' \overline{v''}\,\overline{w_v} &\sqsubseteq v \overline{u''} w_u \\
\mathrm{nf}(v' \overline{w_v}) &\sqsubseteq \mathrm{nf}(v \overline{u''} w_u) \\
v' \overline{w_v} &\sqsubseteq \mathrm{nf}(v \overline{u''} w_u) \\
u' w_u &\sqsubseteq \mathrm{nf}(v \overline{u''} w_u) \\
\mathrm{nf}(\overline{u''}\,\overline{u'} u' w_u) &\sqsubseteq \mathrm{nf}(\overline{u} v \overline{u''} w_u) \\
\mathrm{nf}(\overline{u''} w_u) &\sqsubseteq \mathrm{nf}(\overline{u} v \overline{u''} w_u) \\
x_u &\sqsubseteq \mathrm{nf}(\overline{u} v x_u)
\end{aligned}
$$

Task 65. Let s be a cyclically reduced word. Let W be a set of words and $k = \sum_{w \in W} |w|$. Suppose that $s^{k_1}, \ldots, s^{k_p}$ are pairwise disjoint subwords of words in W and that $k_1, \ldots, k_p$ are pairwise different integers. Show that $p \leq \sqrt{4k/|s| + 1}$.