Task sheet 12

20.01.2025

Task 75. Show that a Lyndon word is not bordered.

Task 76. Give an algorithm that, given a word w and an order on the alphabet, computes the Lyndon word conjugate to w.

Task 77. Show that a word w is bordered if and only if it is a subword of u^k for some word u, where $|u| < |w|$, and some $k \geq 2$.

Task 78. Show that given a solution s to a system of word equations

$$X_0^k = X_1^k X_2^k \cdots X_n^k \text{ for } k = k_1, k_2, k_3 \tag{1}$$

by changing the alphabet $\Gamma \subseteq \mathbb{R}$ we obtain a different solution such that $s(X_0)$ is a 0-word.

Task 79. Show that if the system (1) has a nonperiodic solution then it has a nonperiodic solution over a binary alphabet.

Task 80. Show that the set of 0-sum words are a code.

Task 81. Show the remaining cases of the proof that system of equations (1) not shown in the lecture. The claim is that the number of occurrences for k_2 is not more than the number of occurrences for k_1.
We have covered the case when $\sum(s(X_i)) = 0$ and $\sum(s(X_1 \cdots X_{i-1})) = 0$.

Task 82. Let $u = v$ be a constant-free equation with n unknowns over the alphabet Σ with $|\Sigma| \geq 2$ and $(T_1, \ldots, T_n)$ be a parametric solution. Let $(V_1, \ldots, V_n) \in (\Delta^*)^n$ be the n-tuple obtained from $(T_1, \ldots, T_n)$ by assigning fixed values to all numerical parameters. Show that $(V_1, \ldots, V_n)$ is a solution of the equation $u = v$ over Δ.