Task sheet 13

26.01.2025

Task 83. Show that

$$G_k ab G_{k-1} = G_{k-1} ba G_k$$

For odd k.
To this end show that

$$G_k = F_{k-1} G_{k-2}$$
$$G_k = F_{k-2} G_{k-1}$$

The first follows from definition and the second follows by easy inductive proof.

Task 84. Show that the deterministic register automata are weaker than the non-deterministic ones. Take into the account the number of registers: ideally, you should show that there is a language recognised by 1-register nondeterministic automaton and not by k-register deterministic one.
It is enough to consider the logic using (boolean combinations of) equality tests between read letter and registers.

Hint: Language of words such that the last letter has occurred before seems a good candidate.

Task 85. Show that the symbolic automata are determinizable, i.e. given a non-deterministic symbolic automaton we can construct a deterministic one recognizing the same language.
Conclude that parametric automata are determinizable.

Task 86. Show that the hierarchy of languages recognized by k-parametric automata is strict, i.e. that for each k there is a language recognized by k-parametric automaton but not by any $k-1$-parametric automaton.
A language

$$\bigcup_{a_1,\ldots,a_k} \{a_1, \ldots, a_k\}^*$$

is a good candidate.

Task 87. Show that the hierarchy of languages recognized by k-register automata is strict, i.e. that for each k there is a language recognized by deterministic k-register automaton but not by any $k-1$-register automaton. A language

$$\bigcup_{a_1,\ldots,a_k} (a_1 \cdots a_k)^*$$

is a good candidate.

Task 88. Show there are languages recognized by deterministic 1-counter automaton and not by parametric automata (with any number of parameters).
Consider the logic using (boolean combinations of) equality tests (between read letter and registers/parameters).
Note that there is a subtlety that the parameters are guessed at the beginning but it could in principle hold that, say, for two accepted words w, ww' they are accepted for different parameters values.

Hint: A language of words $\bigcup_a (aa)^*$ is one of good candidates.