GandALF — Exercise Sheet 2

Let $\Sigma = \{a, b, c, d\}$ and L be the language of all words w such that $inf(w) \cap \{a, b\} = \emptyset$ or $c \in inf(w)$.

Exercise 1. Give a Büchi automaton recognizing L.

Exercise 2. Give a deterministic Muller automaton recognizing L.

Exercise 3. Give a deterministic Street automaton recognizing L.

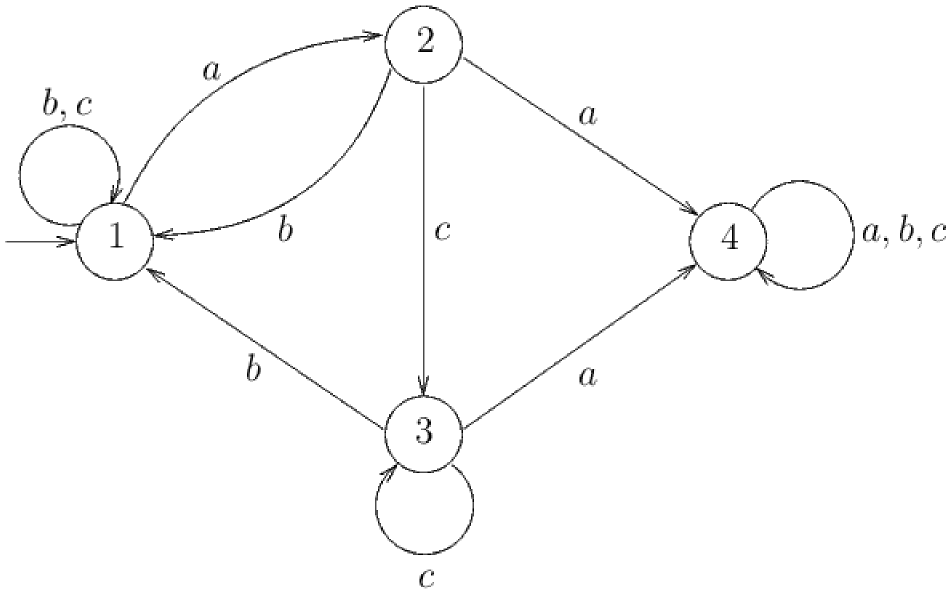
Exercise 4. Give a deterministic Rabin automaton recognizing L.

Exercise 5. Is there a deterministic Büchi automaton recognizing L?

Exercise 6. Is there a deterministic coBüchi automaton recognizing L?

Exercise 6. This exercise was made by Barbara Morawska.

Let $\Sigma := \{a, b, c\}$. From the transition system



we derive four Muller-automata $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$ and $\mathcal{A}_4$ by selecting the sets $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$, $\mathcal{F}_4$ of set of final states as follows:

- $\mathcal{F}_1 := \{\{1, 4\}, \{4\}\}$,
- $\mathcal{F}_2 := \{\{1, 2\}, \{3\}\}$,
- $\mathcal{F}_3 := \{\{1, 2, 3\}\}$ and
- $\mathcal{F}_4 := \{\{1\}, \{1, 2\}, \{3\}, \{1, 2, 3\}\}$.

Determine the ω-languages $L_\omega(\mathcal{A}_1)$, $L_\omega(\mathcal{A}_2)$, $L_\omega(\mathcal{A}_3)$ and $L_\omega(\mathcal{A}_4)$.

The following three exercises are from the Ong's book.

1.9 Prove that an ω-language is deterministic Büchi-recognisable iff it is of the form $\lim U$ for some regular U.

1.11 Consider the ω-language

$$L := \{\alpha \in \{0,1\}^\omega \mid \alpha \text{ contains 00 infinitely often, but 11 only finitely often}\}.$$

(a) Construct a Büchi automaton that recognises L. Explain why it works.

(b) Show that L is not recognisable by a deterministic Büchi automaton.

(c) We say that a ω-automaton *co-Büchi recognises* an ω-word α if there is a run ρ of the automaton on α such that from some point onwards, only final states will be visited i.e. there is an $n \geq 0$ such that for every $i > n$, $\rho(i)$ is a final state.

Is L recognisable by a deterministic co-Büchi automaton? Justify your answer.

1.12

(a) Let L be an ω-language over the alphabet Σ. Define *right-congruence* $\sim_L \subseteq \Sigma^* \times \Sigma^*$ by

$$u \sim_L v \ := \ \forall \alpha \in \Sigma^\omega . u\,\alpha \in L \leftrightarrow v\,\alpha \in L.$$

Prove that every deterministic Muller automaton that recognises L needs at least as many states as there are $\sim_L$-equivalence classes.

Show that there is a ω-language L, which is not ω-regular, such that $\sim_L$ has only finitely many equivalence classes.

Hence, or otherwise, state (without proof) a result about regular $*$-languages (i.e. sets of finite words) that does not generalise to ω-regular ω-languages.

(b) Is it true that an ω-language is ω-regular if and only if it is expressible as a Boolean combination of languages of the form $\lim U$ where U is a regular $*$-language? Justify your answer.

Exercise 10. Given an non-deterministic Büchi automaton on words A, show that there is an equivalent non-deterministic Muller automaton recognising the same language.

Exercise 11. Given a deterministic Muller automaton A, show that there is an non-deterministic Büchi automaton recognising the same language.

Exercise 12. Prove that deterministic Muller automata are closed under complement, i.e. for every DMW A there exists a DMW B with $L(A) = \overline{L(B)}$.