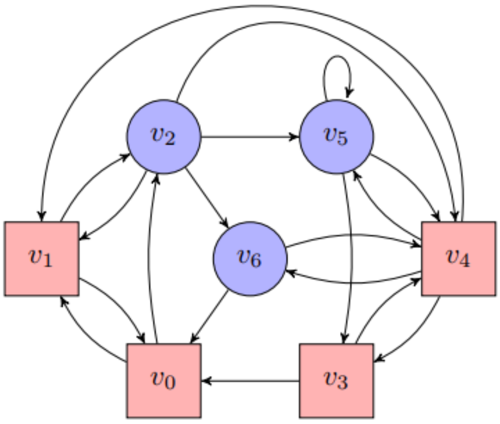
GandALF — Exercise Sheet 4

Exercise 1. (By M. Zimmermann)
Consider the Muller game $\mathcal{G}_1 = (\mathcal{A}_1, \textsc{muller}(\mathcal{F}_1))$ with $\mathcal{A}_1$ as depicted below and

$$\mathcal{F}_1 = \{\{v_0, v_1\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_4, v_5, v_6\}, \{v_0, v_1, v_2, v_6\}\}.$$



Determine the winning regions of $\mathcal{G}_1$ and uniform finite-state winning strategies for both players. Specify the strategies by giving a memory structure (not necessarily the same for both players) and a next-move function.

Exercise 2. (By M. Zimmermann)
A family $\mathcal{F} \subseteq 2^V$ of sets is union-closed, if $F \cup F' \in \mathcal{F}$ for all $F, F' \in \mathcal{F}$. A Muller game is doubly union-closed, if $\mathcal{F}$ and $2^V \setminus \mathcal{F}$ are union-closed.

Show that doubly union-closed Muller games are equivalent to parity games, i.e.

- for every doubly union-closed Muller game $(\mathcal{A}, \textsc{muller}(\mathcal{F}))$ there exists a parity game $(\mathcal{A}, \textsc{parity}(\Omega))$ such that $\textsc{muller}(\mathcal{F}) = \textsc{parity}(\Omega)$ and
- for every parity game $(\mathcal{A}, \textsc{parity}(\Omega))$ there exists a doubly union-closed Muller game $(\mathcal{A}, \textsc{muller}(\mathcal{F}))$ such that $\textsc{parity}(\Omega) = \textsc{muller}(\mathcal{F})$.

Exercise 3. Show that there are games that are not determined.

Exercise 4. (This is an open-ended exercise with multiple correct solutions.) Omega-regular languages are closed under union and intersection. It is much less clear what is an intersection and an union of games. Your task here is to propose a definition of these two operations on games and discuss whether:

- If both games are determined, then their union and intersection are determined.
- If both games are determined with memoryless strategies, then their union and intersection are determined with memoryless strategies.

Exercise 5. Give a non-trivial example of a game such that

- Player 0 wins forgetful on each $\{u\}$ for $u \in W_0$ (winning region of Player 0) and
- Player 1 wins on every $\{u\}$ for $u \notin W_0$, but does not win forgetful on any set.

Exercise 6. (Exercise 2.10 from [2]) Prove that for a finite arena, the winning regions of a Büchi game can be computed in time $O(n(m + n))$, where m is the number of edges and n is the number of nodes.