GandALF — Exercise Sheet 8

Exercise 1. A generalized Büchi automaton is a tuple $\mathcal{A} = (\Sigma, Q, Q_0, \delta, F_1, \dots, F_k)$ such that for every i, the tuple $(\Sigma, Q, Q_0, \delta, F_i)$ is a Büchi automaton. A run π of $\mathcal{A}$ is accepting if and only if π visits infinitely often at least one state in each F_i (i.e., $\forall 1 \leq i \leq k$ we have $Inf(\pi) \cap F_i \neq \emptyset$. What is the expression power of generalized Büchi automata? What is the complexity of the emptiness and the universality problems for generalize Büchi automata?

Exercise 2. Generalized Büchi acceptance conditions can be extended to *generalized Büchi objectives* in two-player games. What is the complexity of solving two-player games with generalized Büchi objectives? What are the memory requirements for both players?

Exercise 3. Prove that NPT are closed under intersection.

Exercise 4. We consider mean-payoff Büchi games (A, w, F), where $A = (V_1, V_2, E)$ is a game arena, $w\colon E \to \{-W, \dots, W\}$ are weights and $F \subseteq V_1 \cup V_2$. In these games, the objective of Player 1 is to simultaneously satisfy the mean-payoff objective $MP_w \leq 0$ (mean-payoff with threshold 0) and the Büchi objective F. Present an algorithm for solving mean-payoff Büchi games. Discuss your algorithm complexity. What are the memory requirements in these games?

Exercise 5. Consider multi-mean-payoff games (A, w_1, w_2), where $A = (V_1, V_2, E)$ is a game arena, $w_1, w_2\colon E \to \{-W, \dots, W\}$ are weights. The objective of Player 1 is to simultaneously satisfy the mean-payoff objective $MP_{w_1} > 0$ and $MP_{w_2} > 0$. Present an algorithm for solving these games. What are the memory requirements in these games?