GandALF — Exercise Sheet 5

Exercise 1. Prove Lemma 6.3 (3) from slides.

Exercise 2. Prove Lemma 6.4 (4) from slides.

Exercise 3. Show that there are games that are not determined.

Exercise 4. For a finite language $L \subseteq \{0,1\}^*$, let L^y $\{vy^\omega \mid v \in L\} \cup \{vn^\omega \mid v \notin L\}$ be a language over the alphabet $\{0,1,y,n\}$. Prove or disprove the following statement:

If L is not regular, then there is a finite game with the winning condition L^y and a vertex v such that Player 1 has a winning strategy on $\{v\}$, but has no forgetful winning strategy on $\{y\}$.

Exercise 5. Assume $w = \binom{1}{0}\binom{0}{0}\binom{0}{0}\binom{0}{0}\left(\binom{0}{0}\binom{0}{1}\right)^\omega$. Which of the following hold?

- $w, 0 \models p_0 \vee p_1$
- $w, 0 \models p_0 \wedge Xp_1$
- $w, 0 \models p_0 U p_1$
- $w, 0 \models \neg p_1 U X(p_1 \wedge X \neg p_1)$

Exercise 6. Let $F\varphi := \top U \varphi$ and $G\varphi := \neg F \neg \varphi$. Which of the following formulas are equivalent?

- GFp_0
- FGp_0
- $Gp_0 \Rightarrow Fp_1$
- $p_0 U (p_1 \vee \neq p_0)$
- $Gp_0 \wedge FGp_0$
- Gp_0

Exercise 7. Let $w \models \varphi$ if for all i we have $w, i \models \varphi$. Prove or disprove the following statement: For every word w and a formula φ, $w \models \varphi$ if and only if $w \not\models \neg\varphi$.

Two languages L_1, L_2 of infinite words are *distinguishable* by an LTL formula φ if there are $w_1 \in L_1$, $w_2 \in L_2$ such that $w_1, 0 \models \varphi$ iff $w_2, 0 \not\models \varphi$.

Exercise 8. Give an example of two different languages not distinguishable by any LTL formula.

Exercise 9. Is there an example of two different ω-regular languages not distinguishable by any LTL formula?

Exercise 10. Is the language $\{w \mid w, 0 \models \varphi\}$ ω-regular for every LTL formula φ?

Exercise 11. Can every ω-regular language be represented as $\{w \mid w, 0 \models \varphi\}$ ω for some LTL formula φ?

Exercise 12. Determine the complexity of the following problem: given a LTL formula φ, is there a word w such that $w, 0 \models \varphi$?