GandALF — Exercise Sheet 9

Exercise 1. Let Σ be the set of agents and $\Gamma \subseteq \Sigma$. Consider the following ATL formulas:

(1) $\langle\langle\Gamma\rangle\rangle X p \iff \neg\langle\langle\Sigma \setminus \Gamma\rangle\rangle X \neg p$,

(2) $\langle\langle\Gamma\rangle\rangle F(p \vee q) \iff \langle\langle\Gamma\rangle\rangle F(p) \vee \langle\langle\Gamma\rangle\rangle F(q)$,

(3) $\langle\langle\Gamma\rangle\rangle F(p \wedge q) \iff \langle\langle\Gamma\rangle\rangle F(p) \wedge \langle\langle\Gamma\rangle\rangle F(q)$, and

(4) $\langle\langle\Gamma\rangle\rangle F(\langle\langle\Gamma\rangle\rangle F p) \iff \langle\langle\Gamma\rangle\rangle F p$.

Which of the above formulas are tautologies? Consider all four semantics: perfect vs. imperfect information, and perfect vs. imperfect recall.

Remark. Let $\mathcal{A}$ be a weighted automaton. We have defined the value of a word w as the infimum over the values of all accepting runs of $\mathcal{A}$ on w. Therefore, if $\mathcal{A}$ has no accepting runs on w, the value of w is the infimum of the empty set, which is ∞.

Exercise 2. Present algorithms deciding the emptiness and the universality problems for weighted automata with the LimSup value function. Recall that, if $\vec{a} = a_1, a_2 \dots$ is a sequence of values from a finite set A, then $\textsc{LimSup}(\vec{a})$ is the maximal value from A that appears infinitely often in $\vec{a}$.

Exercise 3. Construct a weighted automaton $\mathcal{A}$ over the Sum value function such that $\mathcal{A}$ has accepting runs only over words from $(\#a^*)^*$, and for all $w \in (\#a^*)^*$, which are of the form $w = \#a^{x_1}\#a^{x_2}\dots\#a^{x_k}\#$ we have: $\mathcal{L}_{\mathcal{A}}(w) = 0$ if for all $1 \leq i < k$, the absolute value of $x_i - x_{i+1}$ is less or equal to 1, and $\mathcal{L}_{\mathcal{A}}(w) \leq -1$ otherwise.

Exercise 4. Consider a function $f\colon \{a, \#\}^\omega \to \mathbb{R}$ such that $f(w) < \infty$ if $w \in (\#a^*)^\omega$, and $f(\#a^{x_1}\#a^{x_2}\dots) = \textsc{LimAvg}(x_1 x_2 \dots)$. If f can be expressed by a weighted automaton with the LimAvg value function, construct such an automaton. Otherwise, show that it is impossible.

Exercise 5. Show that f_3 defined for every w as the edit distance of w from $L = a^*b^*$ cannot be computed by a deterministic weighted automaton with the Sum value function.

Exercise 6. Present Karp's algorithm computing the minimal mean cycle (prove its correctness and present complexity analysis).