# Tree pattern queries: learning and teaching

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Trees and tree patterns (twigs)



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# Trees and tree patterns (twigs): embeddings



(c) Embeddings of  $q_0$  in  $t_0$ .

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Unary tree patterns and decorated trees



(d) Unary path query  $p_0$ . (e) Decorated trees  $t_1$  and  $t_2$ .

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characterizing queries for a given q provide a (polynomial) set of examples  $CS_q$  such that for all p it holds  $CS_q \subseteq \mathcal{L}(p)$ iff  $q \subseteq p$ .

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Here, we focus on samples with positive examples only.

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Minimality vs. completeness?

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moreover: exponential samples required

Arbitrary twigs require exponential samples

#### Theorem

For any natural number n there exists a twig query q such that any set characterizing q contains at least  $2^n$  examples.

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#### Theorem

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#### Proof.

Construct a query q and a set of twig queries U such that:

- 1. for each  $p \in U$  we have  $q \notin p$ ;
- 2. no single positive example t can witness the fact that any two distinct  $p_1, p_2 \in U$  are not equivalent to q;

3. U contains  $2^n$  queries.

Arbitrary twigs require exponential samples



(g) Query  $p_v$  for  $v = (k_1, ..., k_n)$ .

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#### Anchored tree patterns: a good class

A twig query is *anchored* if:

- 1. A //-edge can be incident to a  $\star\text{-node only}$  if the node is a leaf.
- 2. A \*-node may be a leaf only if it is either incident to a //-edge or it is the selecting node.

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### Anchored tree patterns: a good class

#### Theorem

For anchored twigs:

- *P1.* subsumption  $\equiv$  containment;
- P2. a small characteristic sample always exists (two trees are enough!).

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# P2: Containment characterizing trees



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#### Lemma

Take any anchored query q and construct  $t_1^q$  as above. For any p of height bounded by the height of q and not using labels  $a_1$  and  $a_2$ ,  $t_1^q \leq p$  implies  $q \leq p$ .

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# P2: Containment characterizing trees



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# Corollary If both $t_0^q$ and $t_1^q$ satisfy p, then $q \subseteq p$ .

**Input:** a sample S of decorated trees **Output:** a minimal unary anchored path query p such that  $S \subseteq \mathcal{L}(p)$ 

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 $w := \min_{\leq_{can}} (SelPath(S))$ let w be of the form  $r/a_1/\cdots/a_n$  p := r//\*foreach subpath u of  $a_1/a_2/\cdots/a_{n-1}$ in the order of decreasing lengths **do**replace in p any //-edge by //u// as long as  $S \subseteq \mathcal{L}(p)$ 

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**let** *p* be of the form 
$$r//p_0//b_1$$
  
**if**  $S \subseteq \mathcal{L}(p\{b_1 \leftarrow a_n\})$  **then**  
 $p \coloneqq p\{b_1 \leftarrow a_n\}$ 

foreach descendant edge  $\alpha$  in p do find maximal  $\ell$  s.t.  $S \subseteq \mathcal{L}(p\{\alpha \leftarrow //(*/)^{\ell}\})$ if  $S \subseteq \mathcal{L}(p\{\alpha \leftarrow /(*/)^{\ell}\})$  then  $p \coloneqq p\{\alpha \leftarrow /(*/)^{\ell}\}$ 

return p

A sample and the constructed queries.

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#### Theorem

Our algorithm is sound and complete (i.e., it is sound and returns q if the input is S such that  $S \supseteq CS_q = \{t_0^q, t_1^q\}$ ).

#### Proof.

• Assume some *p* is returned for a given sample *S*.

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- Obviously,  $S \subseteq \mathcal{L}(p)$ .
- By case analysis: there is no  $p' \neq p$  such that  $p' \subseteq p$  and  $S \subseteq \mathcal{L}(p')$ .

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- By case analysis: there is no  $p' \neq p$  such that  $p' \subseteq p$  and  $S \subseteq \mathcal{L}(p')$ .
- Since CS<sub>q</sub> ⊆ S ⊆ L(p) then q ⊆ p (P2). Hence, by the claim above, p and q are equivalent.

### Learning anchored queries

Our algorithm can be extended for

- boolean anchored path queries,
- conjunctions of anchored path queries,
- (path-subsumption-free) boolean twigs,

(path-subsumption-free) unary twigs.